Research Article

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Supporting Web Material: Propriety results and additional simulation study.

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1. Propriety results

In the material below, equation numbers refer to equations in the main paper. Equations associated to the Supporting Web Material have an "S" suffixed to the equation number.

Proposition 1 *Consider the model (3)–(4). Assume that f in (2) is a scale mixture of normals and that $\text{rank}(\mathbf{X}) = p$. Then, the posterior of $(\boldsymbol{\beta}, \omega, \lambda, \delta)$ is proper if the posterior distribution associated to model (3), with errors distributed according to a scale mixture of normals f with shape parameter δ, scale parameter ω, and prior structure given by:*

$$\pi(\boldsymbol{\beta}, \omega, \delta) \propto \frac{\tilde{p}(\delta)}{\omega^a}, \tag{1S}$$

where $\tilde{p}(\delta)$, the marginal prior on δ, is proper.

Corollary 1 *Consider the model (3)–(4) and assume that f in (2) is a scale mixture of normals. Then,*

(i) If $a = 1$, a sufficient condition for the propriety of the posterior of $(\boldsymbol{\beta}, \omega, \delta, \lambda)$ is $n > p$.

(ii) If $a > 1$, a sufficient condition for the propriety of the posterior is $n > p + 1 - a$ and

$$\int \tau^{-\frac{a-1}{2}} \tilde{p}(\delta) dH(\tau|\delta) d\delta < \infty,$$

where $\tilde{p}(\delta)$ represents the marginal prior on δ.

Theorem 1 *Suppose that $T_j|\mathbf{x}_j, \boldsymbol{\beta}, \omega, \lambda, \delta \sim LSS(\mathbf{x}_j^\top \boldsymbol{\beta}, \omega, \lambda, \delta; f, g)$, that f in (9) is a scale mixture of normals, and that $\text{rank}(\mathbf{X}) = p$. Consider the prior structure (4) for this model. Suppose also that $n_c \leq n$ observations are censored and $n_o = n - n_c$ are observed. Then,*

(i) If $a = 1$, a sufficient condition for the propriety of the corresponding posterior is $n_o > p$.

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(ii) If $a > 1$, a sufficient condition for the propriety of the posterior is $n_o > p + 1 - a$ and (5).

An extreme case arises when all of the observations are censored. The following result provides conditions for the propriety of the posterior under this scenario.

Theorem 2 *Consider the model (8) with prior (4), where f is a scale mixture of normals. Suppose that $n_I \leq n$ observations are interval censored, where the length of these intervals is finite, and that the other $n - n_I$ observations are censored (not necessarily in a finite interval). Denote the n_I interval-censored observations as $(I_1, \dots, I_{n_I})$, and let $\mathbf{X}_{n_I}$ be the corresponding design submatrix. Then, the corresponding posterior is proper if $\mathcal{E} = I_1 \times \cdots \times I_{n_I}$ and the column space of $\mathbf{X}_{n_I}$ are disjoint, together with one of the following conditions*

(a) $a = 1$ and $n_I > p$.

(b) $a > 1$, $n_I > p + 1 - a$, and (5).

Theorem 2 represents a generalisation of Theorem 5 from [1] and also provides more tangible conditions on the type of censoring required to guarantee the existence of the posterior distribution. An alternative way of checking that $\mathcal{E}$ and the column space of $\mathbf{X}_{n_I}$ are disjoint consists of formulating this condition as a linear programming (LP) problem. Denote $\boldsymbol{\eta} \in \mathbb{R}^p$, $\boldsymbol{\psi} = (\psi_1, \dots, \psi_{n_I})^\top \in \mathcal{E}$ and $I_j = [l_j, u_j]$, $j = 1, \dots, n_I$. Define the LP problem:

$$\begin{aligned} \text{Find} \quad & \max_{\boldsymbol{\eta}, \boldsymbol{\psi}} \; 1, \\ \text{Subject to} \quad & \mathbf{X}_{n_I} \boldsymbol{\eta} = \boldsymbol{\psi}, \\ \text{and} \quad & \log l_j \leq \psi_j \leq \log u_j, \;\; j = 1, \dots, n_I. \end{aligned} \tag{2S}$$

Thus, condition (iii) is equivalent to verifying the infeasibility of the LP problem (2S), for which there are several theoretical and numerical tools (LP solvers) available (see *e.g.* [2]). Note also that the aim of this formulation is not to solve the maximisation problem (which is trivial), but to check the feasibility of the restrictions. This is if the LP solver reports infeasibility, then the sets are disjoint, otherwise it should return some feasible vector.

2. Extensions to multivariate linear regression

In this section, we present a multivariate extension of the Bayesian LRM discussed in Section 2. Recall first that a random vector $\mathbf{Z}$ is said to be distributed according to a $m-$variate skew-symmetric distribution, $m \geq 1$, if its PDF can be written as [3]:

$$s(\mathbf{z}|\boldsymbol{\xi}) = 2f(\mathbf{z} - \boldsymbol{\xi})\varphi(\mathbf{z} - \boldsymbol{\xi}), \;\; \boldsymbol{z} \in \mathbb{R}^m, \tag{3S}$$

where f is a symmetric PDF with support on $\mathbb{R}^m$, and $\varphi : \mathbb{R}^m \to [0, 1]$ satisfies the condition $\varphi(\mathbf{z}) = 1 - \varphi(-\mathbf{z})$. This class of distributions contains the multivariate skew-normal [4] and the multivariate skew-t distributions [5] as particular cases.

Consider now the linear regression model:

$$\mathbf{y}_j = \mathbf{B}^\top \mathbf{x}_j + \boldsymbol{\varepsilon}_j, \;\; j = 1, \dots, n, \tag{4S}$$

where $\mathbf{y}_j \in \mathbb{R}^m$, $\mathbf{B}$ is a $p \times m$ matrix of regression parameters, and $\mathbf{x}_j$ is a known $p \times 1$ vector of covariates. Let $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)^\top$ denote the entire design matrix, and suppose that this matrix has full column rank. We focus on the study of the model (4S) with errors distributed according to a certain class of skew-symmetric distributions. In order

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to introduce this model, recall that a PDF f is said to be a multivariate scale mixture of normals (MSMN), if it can be written as follows:

$$f(\mathbf{z}|\boldsymbol{\xi},\boldsymbol{\Sigma},\delta) = \int_0^\infty \frac{\tau^{m/2}}{(2\pi)^{m/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{\tau}{2}\left(\mathbf{z}-\boldsymbol{\xi}\right)^\top \boldsymbol{\Sigma}^{-1}\left(\mathbf{z}-\boldsymbol{\xi}\right)\right\} dH(\tau|\delta), \quad \mathbf{z}\in\mathbb{R}^m,$$

where H is a mixing distribution, $\delta \in \Delta \subset \mathbb{R}$ is a shape parameter, and $\boldsymbol{\Sigma}$ is a $m \times m$ positive definite symmetric matrix. This family contains the multivariate normal and the multivariate Student-t distributions, among others. We say that a density s is a multivariate skew-symmetric scale mixture of normals (MSSSMN) if it can be written as in (3S) where f is a MSMN.

Theorem 3 *Consider the linear regression model (4S) and suppose that the errors* $\boldsymbol{\varepsilon}_j \stackrel{i.i.d.}{\sim}$ *MSSSMN*$(\mathbf{0},\boldsymbol{\Sigma},\boldsymbol{\lambda},\delta; f,\varphi)$*, where* f *is a MSMN with shape parameter* δ*, and the skewing function* φ *contains a skewness parameter vector* $\boldsymbol{\lambda}$*. Adopt the prior structure:*

$$\pi(\boldsymbol{\beta},\boldsymbol{\Sigma},\boldsymbol{\lambda},\delta) \propto \frac{p(\boldsymbol{\lambda},\delta)}{\det(\boldsymbol{\Sigma})^{\frac{m+1}{2}}}, \tag{5S}$$

where $p(\boldsymbol{\lambda},\delta)$ *is assumed to be proper. Then, the posterior distribution is proper, for almost any sample, provided that* $n \geq m+p$ *and* $\text{rank}(\mathbf{X}) = p$*.*

This result indicates that the introduction of skewness, via the skew-symmetric construction (3S), does not affect the existence of the posterior, provided that the prior on the skewness parameter vector $\boldsymbol{\lambda}$ is proper. The prior structure (5S) is of interest in practice given that it is a generalisation of the structure of the independence Jeffreys prior (see [6] for the structure of this prior in the symmetric case). Moreover, it provides a general structure that leads to proper posteriors under rather mild conditions. Analogously to the results presented for univariate responses, this result holds with probability one. We refer the reader to [6] for a discussion on some zero-probability events that produce improper posteriors. The following results present particular details on the propriety of the posterior for skew-normal and skew-t errors

Corollary 2 *Consider the linear regression model (4S) and suppose that the errors are distributed according to the multivariate skew-normal distribution [4]. Adopt the prior structure:*

$$\pi(\boldsymbol{\beta},\boldsymbol{\Sigma},\boldsymbol{\lambda}) \propto \frac{p(\boldsymbol{\lambda})}{\det(\boldsymbol{\Sigma})^{\frac{m+1}{2}}},$$

where $p(\boldsymbol{\lambda})$ *is assumed to be proper. Then, the posterior distribution is proper provided that* $n \geq m+p$ *and* $\text{rank}(\mathbf{X}) = p$*.*

Corollary 3 *Consider the linear regression model (4S) and suppose that the errors are distributed according to the multivariate skew-*t *distribution [5] with unknown degrees of freedom* $\delta > 0$*. Adopt the prior structure:*

$$\pi(\boldsymbol{\beta},\boldsymbol{\Sigma},\boldsymbol{\lambda},\delta) \propto \frac{p(\boldsymbol{\lambda})p(\delta)}{\det(\boldsymbol{\Sigma})^{\frac{m+1}{2}}},$$

where $p(\boldsymbol{\lambda})$ *and* $p(\delta)$ *are assumed to be proper. Then, the posterior distribution is proper provided that* $n \geq m+p$ *and* $\text{rank}(\mathbf{X}) = p$*.*

Proofs

Proof of Proposition 1

The marginal likelihood of the data is given by

$$m(\mathbf{y}) = \int \left[\prod_{j=1}^{n} s(y_j | \mathbf{x}_j^\top \boldsymbol{\beta}, \omega, \delta, \lambda) \right] \frac{p(\lambda, \delta)}{\omega^a} d\boldsymbol{\beta} d\omega d\delta d\lambda.$$

Using that $0 \leq G(\cdot) \leq 1$ in (2) and that $p(\lambda, \delta)$ is proper it follows that

$$\begin{aligned} m(\mathbf{y}) &\leq \int \left[\prod_{j=1}^{n} \frac{2}{\omega} f\left(\frac{y_j - \mathbf{x}_j^\top \boldsymbol{\beta}}{\omega} \middle| \delta \right) \right] \frac{p(\lambda, \delta)}{\omega^a} d\boldsymbol{\beta} d\omega d\delta d\lambda \\ &= 2^n \int \left[\prod_{j=1}^{n} \frac{1}{\omega} f\left(\frac{y_j - \mathbf{x}_j^\top \boldsymbol{\beta}}{\omega} \middle| \delta \right) \right] \frac{\tilde{p}(\delta)}{\omega^a} d\boldsymbol{\beta} d\omega d\delta, \end{aligned} \quad (6S)$$

where $\tilde{p}(\delta) = \int p(\lambda, \delta) d\lambda$, which is proper. The result follows by noting that the last term in inequality (6S) corresponds to the marginal likelihood of a LRM with residual errors distributed according to the symmetric distribution f and prior structure $\pi(\boldsymbol{\beta}, \omega, \delta) \propto \tilde{p}(\delta)/\omega^a$. □

Proof of Corollary 1

(i) follows by Proposition 1 together with Theorem 1 in [7]. (ii) follows by Proposition 1 together with the proofs of Lemma 1 from [6] and Theorem 2 from [1]. □

Proof of Theorem 1

As discussed in [1], the contribution of a censored observation to the likelihood function is a factor in $[0, 1]$. Then, the marginal likelihood of the complete sample is upper bounded by the marginal likelihood of the uncensored observations. Therefore, the propriety of the posterior distribution can be based on the uncensored observations. □

Proof of Theorem 2

Provided that either (i) or (ii) are satisfied, it follows that

$$I(t_1, \dots, t_{n_I}) = \int \prod_{j=1}^{n_I} s_l(t_j | \mathbf{x}_j^\top \boldsymbol{\beta}, \omega, \lambda, \delta) \pi(\boldsymbol{\beta}, \omega, \lambda, \delta) d\boldsymbol{\beta} d\omega d\lambda d\delta < \infty,$$

for each $(t_1, \dots, t_{n_I}) \in \mathcal{E}$. Finally, given that the Lebesgue measure of $\mathcal{E}$ is finite (since this set is bounded), it follows that the marginal likelihood of the data $\int_{\mathcal{E}} I(t_1, \dots, t_{n_I}) dt_1 \cdots dt_{n_I}$ is finite. □

Proof of Theorem 3

The proof follows analogously to the proof of Proposition 1 by noting that the inequality $2f(\mathbf{x})\varphi(\mathbf{x}) \leq 2f(\mathbf{x})$ holds. □

3. Simulation with skew-logistic and skew-t errors.

For a description of the simulation scenario see Section 4 of the main paper.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.377	4.971	5.607	4.364	4.966	5.596	4.463	4.966	5.512	0.960	–
β_2	1.846	1.998	2.150	1.841	1.999	2.155	1.845	1.998	2.149	0.951	–
β_3	2.862	2.999	3.147	2.862	3.001	3.154	2.861	2.999	3.148	0.960	–
ω	0.442	0.523	0.666	0.458	0.529	0.627	0.476	0.553	0.652	0.904	–
λ	-1.114	0.037	1.194	-0.720	0.026	0.709	-0.861	0.038	0.895	0.961	2.817
$\lambda = 1$											
β_1	4.684	4.993	5.484	4.695	5.020	5.530	4.745	5.065	5.506	0.959	–
β_2	1.884	2.001	2.110	1.881	2.004	2.121	1.880	2.001	2.111	0.958	–
β_3	2.886	3.003	3.122	2.884	3.005	3.131	2.884	3.004	3.123	0.954	–
ω	0.381	0.493	0.654	0.387	0.469	0.625	0.405	0.490	0.627	0.979	–
λ	0.014	1.086	2.924	-0.053	0.593	2.006	-0.044	0.816	2.305	0.958	1.390
$\lambda = 2$											
β_1	4.812	4.996	5.317	4.807	5.011	5.346	4.831	5.050	5.359	0.942	–
β_2	1.896	2.000	2.094	1.893	2.001	2.098	1.896	2.000	2.095	0.959	–
β_3	2.896	2.998	3.092	2.893	2.999	3.091	2.898	2.998	3.091	0.959	–
ω	0.367	0.496	0.617	0.350	0.467	0.606	0.372	0.474	0.605	0.958	–
λ	0.664	2.260	5.121	0.322	1.466	3.443	0.548	1.772	4.236	0.947	0.307
$\lambda = 3$											
β_1	4.859	5.002	5.230	4.854	5.008	5.246	4.872	5.030	5.277	0.920	–
β_2	1.915	2.002	2.087	1.907	2.001	2.088	1.914	2.000	2.088	0.955	–
β_4	2.913	3.000	3.087	2.910	3.000	3.086	2.915	3.000	3.086	0.968	–
ω	0.374	0.492	0.601	0.342	0.476	0.593	0.361	0.478	0.590	0.933	–
λ	1.213	3.225	9.416	0.580	2.305	5.027	0.912	2.660	6.595	0.923	0.078
$\lambda = 4$											
β_1	4.874	4.999	5.156	4.867	5.004	5.182	4.880	5.017	5.206	0.927	–
β_2	1.925	1.997	2.081	1.923	1.998	2.080	1.926	1.997	2.077	0.959	–
β_3	2.921	2.996	3.075	2.919	2.996	3.081	2.924	2.997	3.075	0.961	–
ω	0.398	0.498	0.603	0.369	0.488	0.599	0.385	0.489	0.600	0.938	–
λ	1.991	4.457	107.873	-0.893	3.108	7.313	1.505	3.748	13.127	0.918	0.020
$\lambda = 5$											
β_1	4.892	4.999	5.138	4.885	4.998	5.156	4.899	5.006	5.180	0.929	–
β_2	1.931	2.002	2.078	1.927	2.001	2.075	1.930	2.001	2.075	0.957	–
β_3	2.924	3.001	3.074	2.925	3.001	3.076	2.926	3.001	3.070	0.958	–
ω	0.400	0.499	0.591	0.384	0.489	0.588	0.390	0.495	0.590	0.937	–
λ	2.574	5.741	144.920	-3.665	3.788	12.203	2.031	4.879	18.220	0.919	0.015

Table 1S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-logistic distribution, $n = 100$.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.614	5.004	5.417	4.572	5.010	5.435	4.616	5.016	5.395	0.941	–
β_2	1.911	2.000	2.089	1.903	2.001	2.092	1.910	2.001	2.088	0.959	–
β_3	2.914	3.001	3.098	2.911	3.003	3.100	2.914	3.002	3.100	0.956	–
ω	0.464	0.510	0.587	0.470	0.517	0.575	0.480	0.528	0.589	0.917	–
λ	-0.600	-0.020	0.603	-0.538	-0.020	0.488	-0.595	-0.023	0.573	0.935	4.669
$\lambda = 1$											
β_1	4.784	4.999	5.306	4.786	5.013	5.312	4.820	5.043	5.294	0.969	–
β_2	1.922	2.000	2.070	1.920	2.000	2.076	1.922	2.000	2.070	0.957	–
β_3	2.929	3.001	3.078	2.926	2.999	3.078	2.928	3.000	3.078	0.948	–
ω	0.409	0.499	0.598	0.404	0.474	0.586	0.419	0.487	0.585	0.967	–
λ	0.280	1.028	1.942	0.217	0.785	1.718	0.314	0.885	1.738	0.965	0.405
$\lambda = 2$											
β_1	4.879	5.011	5.182	4.882	5.014	5.187	4.888	5.034	5.218	0.925	–
β_2	1.941	2.001	2.059	1.938	2.000	2.062	1.940	2.001	2.059	0.962	–
β_3	2.941	2.999	3.058	2.940	3.001	3.058	2.940	2.999	3.058	0.961	–
ω	0.413	0.492	0.569	0.389	0.483	0.568	0.402	0.483	0.564	0.941	–
λ	1.060	1.993	3.219	0.777	1.743	2.861	0.910	1.813	2.999	0.922	0.003
$\lambda = 3$											
β_1	4.914	5.003	5.106	4.913	5.004	5.117	4.921	5.012	5.129	0.946	–
β_2	1.949	2.001	2.051	1.948	2.000	2.053	1.950	2.001	2.051	0.963	–
β_3	2.947	2.999	3.051	2.947	2.999	3.056	2.948	2.999	3.052	0.959	–
ω	0.430	0.496	0.565	0.422	0.492	0.562	0.421	0.493	0.561	0.936	–
λ	1.942	3.136	4.910	1.676	2.801	4.328	1.752	2.914	4.584	0.937	1×10^{-8}
$\lambda = 4$											
β_1	4.923	4.999	5.077	4.923	5.001	5.080	4.929	5.004	5.085	0.952	–
β_2	1.954	1.999	2.046	1.953	1.998	2.049	1.955	1.998	2.047	0.955	–
β_3	2.952	2.999	3.046	2.952	2.999	3.046	2.952	3.000	3.046	0.967	–
ω	0.439	0.499	0.560	0.432	0.495	0.556	0.434	0.496	0.559	0.948	–
λ	2.788	4.161	6.653	2.487	3.717	5.715	2.580	3.924	6.203	0.953	2×10^{-11}
$\lambda = 5$											
β_1	4.940	4.999	5.070	4.937	5.000	5.073	4.942	5.002	5.075	0.957	–
β_2	1.958	2.000	2.043	1.956	2.001	2.042	1.958	2.000	2.043	0.965	–
β_3	2.958	2.999	3.041	2.958	2.998	3.042	2.959	2.998	3.041	0.971	–
ω	0.442	0.499	0.556	0.439	0.496	0.555	0.441	0.497	0.555	0.958	–
λ	3.432	5.271	8.539	3.002	4.631	7.160	3.233	4.919	7.875	0.950	2×10^{-11}

Table 2S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-logistic distribution, $n = 250$.

4. Simulation with prior (7)

For a description of the simulation scenario see Section 4 of the main paper.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.730	4.993	5.254	4.721	4.995	5.272	4.715	4.991	5.270	0.926	–
β_2	1.932	1.999	2.065	1.933	1.999	2.067	1.933	1.999	2.065	0.950	–
β_3	2.935	3.002	3.064	2.935	3.002	3.068	2.935	3.002	3.064	0.956	–
ω	0.473	0.504	0.543	0.478	0.509	0.545	0.482	0.514	0.551	0.947	–
λ	-0.331	0.006	0.369	-0.328	0.011	0.328	-0.369	0.008	0.396	0.928	7.415
$\lambda = 1$											
β_1	4.832	5.006	5.235	4.828	5.009	5.226	4.851	5.034	5.226	0.946	–
β_2	1.948	2.000	2.054	1.945	1.999	2.053	1.947	2.000	2.055	0.952	–
β_3	2.946	3.001	3.054	2.945	2.999	3.057	2.947	3.000	3.055	0.943	–
ω	0.429	0.499	0.572	0.422	0.488	0.568	0.434	0.492	0.565	0.948	–
λ	0.431	0.995	1.677	0.364	0.887	1.578	0.460	0.908	1.580	0.946	0.03
$\lambda = 2$											
β_1	4.918	5.003	5.111	4.919	5.005	5.121	4.924	5.013	5.132	0.947	–
β_2	1.954	2.000	2.044	1.952	2.000	2.044	1.954	1.999	2.044	0.943	–
β_3	2.957	2.999	3.042	2.956	2.999	3.043	2.957	2.998	3.042	0.958	–
ω	0.444	0.498	0.546	0.435	0.493	0.545	0.438	0.492	0.543	0.955	–
λ	1.355	2.019	2.740	1.183	1.894	2.628	1.239	1.921	2.653	0.948	2×10^{-16}
$\lambda = 3$											
β_1	4.938	5.002	5.068	4.939	5.004	5.072	4.939	5.006	5.077	0.950	–
β_2	1.964	1.999	2.036	1.960	1.999	2.036	1.963	2.000	2.035	0.961	–
β_3	2.961	2.999	3.037	2.961	2.999	3.039	2.962	2.998	3.036	0.953	–
ω	0.452	0.498	0.542	0.451	0.495	0.543	0.450	0.496	0.543	0.956	–
λ	2.270	2.990	4.101	2.125	2.867	3.857	2.161	2.910	3.960	0.950	4×10^{-40}
$\lambda = 4$											
β_1	4.948	5.002	5.056	4.948	5.003	5.059	4.949	5.004	5.059	0.950	–
β_2	1.966	2.000	2.034	1.966	2.000	2.034	1.966	2.000	2.033	0.952	–
β_3	2.968	3.001	3.034	2.967	3.001	3.035	2.967	3.000	3.034	0.949	–
ω	0.457	0.499	0.542	0.454	0.498	0.543	0.455	0.498	0.543	0.955	–
λ	3.062	4.081	5.598	2.884	3.846	5.311	2.968	3.961	5.361	0.946	7×10^{-43}
$\lambda = 5$											
β_1	4.958	4.999	5.051	4.957	5.000	5.051	4.958	5.001	5.053	0.958	–
β_2	1.968	1.999	2.028	1.967	1.998	2.028	1.969	1.999	2.028	0.952	–
β_3	2.970	3.001	3.033	2.968	3.001	3.033	2.970	3.001	3.032	0.952	–
ω	0.457	0.498	0.537	0.456	0.497	0.537	0.457	0.497	0.537	0.952	–
λ	3.806	5.143	7.036	3.577	4.828	6.536	3.688	4.982	6.740	0.947	1×10^{-43}

Table 3S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-logistic distribution, $n = 500$.

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Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.587	5.009	5.395	4.560	5.012	5.427	4.603	5.009	5.377	0.951	–
β_2	1.891	1.999	2.109	1.890	1.999	2.112	1.890	1.998	2.108	0.950	–
β_3	2.893	3.002	3.109	2.884	3.002	3.105	2.890	3.001	3.108	0.963	–
ω	0.403	0.525	0.732	0.429	0.557	0.735	0.446	0.584	0.768	0.859	–
λ	-1.064	-0.032	1.096	-0.907	-0.021	0.861	-0.984	-0.026	0.989	0.951	–
δ	1.927	3.187	8.596	1.941	3.128	7.486	2.124	3.754	12.291	0.904	3.166
$\lambda = 1$											
β_1	4.780	4.996	5.262	4.753	4.983	5.269	4.789	5.007	5.273	0.960	–
β_2	1.914	2.001	2.088	1.913	2.001	2.091	1.915	1.999	2.088	0.960	–
β_3	2.917	2.999	3.086	2.911	2.997	3.089	2.917	2.998	3.088	0.959	–
ω	0.364	0.498	0.746	0.380	0.497	0.746	0.397	0.528	0.755	0.944	–
λ	0.198	1.061	2.510	0.071	0.875	2.097	0.123	0.959	2.392	0.963	–
δ	1.918	3.162	10.156	1.917	3.021	6.954	2.095	3.633	11.724	0.914	0.966
$\lambda = 2$											
β_1	4.847	4.994	5.158	4.843	4.980	5.149	4.856	4.993	5.167	0.951	–
β_2	1.933	2.000	2.074	1.928	2.001	2.076	1.930	2.000	2.074	0.950	–
β_3	2.933	2.999	3.072	2.929	2.999	3.074	2.933	2.999	3.073	0.969	–
ω	0.361	0.507	0.717	0.351	0.509	0.728	0.377	0.522	0.725	0.949	–
λ	1.001	2.262	5.496	0.782	1.867	3.979	0.912	2.091	4.776	0.950	–
δ	1.938	3.179	10.717	1.930	2.948	7.047	2.082	3.485	11.200	0.927	0.061
$\lambda = 3$											
β_1	4.883	4.991	5.106	4.880	4.985	5.104	4.893	4.990	5.110	0.952	–
β_2	1.939	2.002	2.067	1.937	2.002	2.066	1.942	2.001	2.064	0.955	–
β_3	2.943	3.000	3.061	2.942	2.999	3.062	2.944	2.999	3.059	0.966	–
ω	0.358	0.511	0.698	0.361	0.512	0.708	0.374	0.525	0.703	0.951	–
λ	1.702	3.509	15.273	1.339	2.780	6.306	1.562	3.205	8.903	0.936	–
δ	1.947	3.212	13.439	1.923	2.968	6.882	2.101	3.494	11.393	0.921	0.015
$\lambda = 4$											
β_1	4.910	4.995	5.084	4.904	4.987	5.080	4.913	4.994	5.086	0.964	–
β_2	1.945	2.002	2.062	1.942	2.002	2.060	1.946	2.002	2.059	0.949	–
β_3	2.949	2.999	3.060	2.944	2.999	3.058	2.949	2.999	3.058	0.960	–
ω	0.368	0.511	0.694	0.371	0.516	0.693	0.383	0.526	0.699	0.957	–
λ	2.214	4.847	29.796	1.805	3.645	8.605	2.096	4.349	13.213	0.957	–
δ	1.954	3.276	12.496	1.890	2.946	6.617	2.063	3.497	10.987	0.931	0.008
$\lambda = 5$											
β_1	4.925	4.997	5.072	4.919	4.991	5.067	4.925	4.994	5.069	0.961	–
β_2	1.948	2.001	2.054	1.948	2.001	2.055	1.950	2.002	2.053	0.964	–
β_3	2.951	3.000	3.055	2.948	3.000	3.052	2.951	3.000	3.050	0.978	–
ω	0.370	0.512	0.686	0.371	0.514	0.688	0.390	0.525	0.684	0.945	–
λ	2.813	5.988	56.575	2.226	4.317	10.681	2.673	5.371	18.051	0.960	–
δ	1.954	3.218	12.533	1.873	2.950	6.665	2.052	3.498	10.688	0.930	0.007

Table 4S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-t distribution with 3 degrees of freedom, $n = 100$.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.776	4.997	5.235	4.749	4.997	5.244	4.780	4.998	5.227	0.964	–
β_2	1.935	2.000	2.064	1.931	1.998	2.067	1.935	1.999	2.065	0.965	–
β_3	2.937	3.000	3.066	2.934	3.000	3.070	2.936	3.000	3.067	0.953	–
ω	0.439	0.509	0.599	0.449	0.523	0.616	0.456	0.531	0.628	0.914	–
λ	-0.526	0.005	0.509	-0.500	0.005	0.490	-0.481	0.006	0.496	0.967	–
δ	2.252	3.040	4.944	2.249	3.022	4.803	2.369	3.195	5.480	0.927	5.582
$\lambda = 1$											
β_1	4.857	5.001	5.157	4.847	4.996	5.164	4.856	5.003	5.160	0.951	–
β_2	1.950	1.998	2.052	1.949	2.000	2.055	1.951	1.998	2.052	0.960	–
β_3	2.945	2.999	3.053	2.944	3.000	3.055	2.945	2.999	3.054	0.954	–
ω	0.408	0.499	0.639	0.407	0.494	0.633	0.420	0.507	0.641	0.946	–
λ	0.461	1.024	1.768	0.401	0.962	1.720	0.430	0.989	1.759	0.954	–
δ	2.244	3.036	4.987	2.224	2.959	4.732	2.316	3.158	5.440	0.940	0.140
$\lambda = 2$											
β_1	4.914	4.999	5.088	4.911	4.997	5.091	4.914	5.000	5.091	0.945	–
β_2	1.960	2.000	2.042	1.958	2.000	2.045	1.960	2.000	2.043	0.961	–
β_3	2.955	2.998	3.042	2.952	2.999	3.042	2.955	2.999	3.042	0.955	–
ω	0.404	0.501	0.615	0.401	0.500	0.617	0.407	0.505	0.625	0.953	–
λ	1.271	2.067	3.226	1.176	1.932	3.017	1.218	2.009	3.154	0.953	–
δ	2.232	3.050	4.992	2.194	2.944	4.575	2.288	3.152	5.214	0.949	8×10^{-9}
$\lambda = 3$											
β_1	4.937	4.999	5.067	4.933	4.997	5.065	4.934	5.000	5.067	0.949	–
β_2	1.966	2.000	2.038	1.963	2.000	2.038	1.965	2.000	2.037	0.953	–
β_3	2.963	2.999	3.034	2.961	2.999	3.036	2.962	2.998	3.033	0.949	–
ω	0.410	0.502	0.604	0.406	0.502	0.614	0.416	0.507	0.612	0.956	–
λ	2.030	3.169	5.009	1.825	2.866	4.571	1.949	3.042	4.807	0.948	–
δ	2.240	3.090	5.062	2.156	2.928	4.647	2.286	3.150	5.227	0.937	2×10^{-12}
$\lambda = 4$											
β_1	4.950	5.000	5.056	4.948	4.997	5.058	4.950	5.000	5.058	0.953	–
β_2	1.969	1.999	2.033	1.968	2.000	2.034	1.970	1.999	2.032	0.952	–
β_3	2.967	2.999	3.031	2.966	2.998	3.032	2.968	2.999	3.032	0.959	–
ω	0.413	0.505	0.596	0.412	0.503	0.600	0.416	0.507	0.603	0.954	–
λ	2.749	4.201	6.931	2.497	3.784	5.931	2.650	4.063	6.461	0.960	–
δ	2.243	3.081	4.863	2.187	2.965	4.488	2.289	3.147	5.086	0.946	6×10^{-12}
$\lambda = 5$											
β_1	4.957	4.999	5.049	4.953	4.998	5.048	4.955	5.000	5.050	0.950	–
β_2	1.973	2.000	2.029	1.971	1.999	2.029	1.972	1.999	2.029	0.955	–
β_3	2.971	2.999	3.030	2.970	2.999	3.030	2.971	2.999	3.029	0.965	–
ω	0.412	0.504	0.591	0.413	0.504	0.599	0.415	0.507	0.598	0.945	–
λ	3.315	5.228	9.096	2.939	4.638	7.558	3.161	4.977	8.352	0.955	–
δ	2.231	3.068	5.116	2.157	2.928	4.503	2.278	3.137	5.264	0.935	7×10^{-11}

Table 5S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-t distribution with 3 degrees of freedom, $n = 250$.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.831	4.990	5.162	4.823	4.992	5.172	4.834	4.991	5.158	0.954	–
β_2	1.951	1.999	2.042	1.951	1.999	2.045	1.951	2.000	2.043	0.954	–
β_3	2.955	3.002	3.046	2.955	3.002	3.047	2.954	3.002	3.047	0.951	–
ω	0.455	0.506	0.565	0.460	0.513	0.575	0.464	0.517	0.579	0.918	–
ω	-0.367	0.024	0.369	-0.362	0.019	0.378	-0.362	0.024	0.369	0.955	–
δ	2.414	3.060	4.103	2.381	3.044	4.040	2.463	3.133	4.269	0.940	8.122
$\lambda = 1$											
β_1	4.890	5.000	5.105	4.887	4.998	5.110	4.894	5.002	5.110	0.942	–
β_2	1.960	2.000	2.035	1.958	2.000	2.037	1.960	1.999	2.035	0.947	–
β_3	2.965	3.000	3.038	2.963	3.000	3.041	2.965	3.000	3.038	0.972	–
ω	0.433	0.501	0.594	0.430	0.499	0.595	0.437	0.505	0.595	0.955	–
ω	0.610	1.015	1.556	0.566	0.984	1.502	0.581	1.002	1.538	0.938	–
δ	2.416	3.058	4.121	2.415	3.020	4.025	2.475	3.118	4.212	0.937	5×10^{-7}
$\lambda = 2$											
β_1	4.938	4.999	5.066	4.931	4.998	5.065	4.937	5.001	5.069	0.941	–
β_2	1.969	1.999	2.028	1.968	1.999	2.029	1.969	1.999	2.028	0.947	–
β_3	2.972	3.000	3.030	2.971	3.001	3.031	2.972	3.000	3.030	0.965	–
ω	0.428	0.504	0.583	0.428	0.502	0.586	0.430	0.504	0.586	0.937	–
ω	1.445	2.032	2.877	1.387	1.962	2.775	1.428	2.001	2.838	0.943	–
δ	2.423	3.068	4.142	2.370	2.988	4.024	2.461	3.105	4.257	0.949	8×10^{-35}
$\lambda = 3$											
β_1	4.957	4.998	5.045	4.955	4.999	5.045	4.957	4.999	5.045	0.960	–
β_2	1.973	1.999	2.024	1.973	1.999	2.025	1.973	1.999	2.024	0.952	–
β_3	2.976	3.000	3.025	2.975	2.999	3.027	2.976	3.000	3.026	0.969	–
ω	0.434	0.504	0.573	0.433	0.504	0.575	0.435	0.506	0.573	0.957	–
ω	2.291	3.080	4.228	2.175	2.968	4.020	2.251	3.031	4.146	0.960	–
δ	2.432	3.056	4.075	2.376	2.999	3.942	2.456	3.081	4.127	0.950	1×10^{-43}
$\lambda = 4$											
β_1	4.964	4.999	5.033	4.963	4.999	5.034	4.964	4.999	5.035	0.958	–
β_2	1.976	1.999	2.021	1.976	1.999	2.022	1.977	1.999	2.021	0.949	–
β_3	2.978	3.000	3.022	2.978	3.000	3.023	2.979	3.000	3.022	0.958	–
ω	0.437	0.503	0.569	0.439	0.505	0.572	0.440	0.506	0.571	0.950	–
ω	3.052	4.112	5.798	2.911	3.915	5.533	2.986	4.018	5.666	0.956	–
δ	2.417	3.043	4.144	2.367	2.974	4.023	2.437	3.085	4.206	0.941	3×10^{-41}
$\lambda = 5$											
β_1	4.970	4.999	5.029	4.969	4.998	5.028	4.970	4.999	5.029	0.963	–
β_2	1.979	1.999	2.019	1.978	1.999	2.020	1.978	1.999	2.019	0.956	–
β_3	2.980	3.001	3.021	2.980	3.001	3.021	2.980	3.001	3.021	0.956	–
ω	0.443	0.503	0.563	0.441	0.502	0.567	0.445	0.505	0.566	0.949	–
ω	3.835	5.221	7.380	3.613	4.913	6.910	3.748	5.094	7.178	0.945	–
δ	2.412	3.066	4.086	2.366	2.985	3.962	2.442	3.094	4.159	0.953	3×10^{-35}

Table 6S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-t distribution with 3 degrees of freedom, $n = 500$.

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Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.477	4.991	5.535	4.617	4.999	5.404	4.802	4.997	5.247	0.999	–
β_2	1.916	2.001	2.085	1.915	2.000	2.088	1.918	2.001	2.082	0.956	–
β_3	2.916	2.997	3.088	2.913	2.997	3.088	2.914	2.997	3.087	0.950	–
ω	0.485	0.606	0.759	0.462	0.523	0.605	0.486	0.550	0.635	0.904	–
λ	-2.079	-0.002	2.032	-0.487	-0.006	0.383	-0.659	0.005	0.568	0.999	1.061
$\lambda = 1$											
β_1	4.790	5.010	5.318	4.870	5.249	5.474	4.969	5.242	5.400	0.915	–
β_2	1.925	1.999	2.071	1.926	1.999	2.073	1.927	1.999	2.071	0.942	–
β_3	2.927	3.001	3.065	2.925	3.002	3.070	2.927	3.002	3.065	0.961	–
ω	0.368	0.487	0.646	0.381	0.436	0.525	0.397	0.458	0.546	0.990	–
λ	0.006	0.995	2.659	-0.306	0.025	1.500	-0.328	0.106	1.164	0.939	1.0321
$\lambda = 2$											
β_1	4.885	5.005	5.290	4.916	5.064	5.423	4.946	5.188	5.368	0.799	–
β_2	1.944	2.000	2.056	1.941	2.000	2.056	1.945	2.000	2.055	0.963	–
β_3	2.942	2.998	3.058	2.940	2.998	3.061	2.943	2.998	3.058	0.957	–
ω	0.342	0.491	0.603	0.321	0.380	0.558	0.340	0.410	0.548	0.838	–
λ	0.240	2.116	5.181	-0.195	0.230	2.779	-0.007	0.672	2.980	0.783	0.749
$\lambda = 3$											
β_1	4.911	4.998	5.115	4.939	5.028	5.401	4.949	5.064	5.339	0.869	–
β_2	1.951	2.000	2.051	1.949	2.001	2.048	1.950	2.001	2.048	0.960	–
β_3	2.944	3.000	3.052	2.943	2.997	3.053	2.946	2.999	3.052	0.952	–
ω	0.394	0.494	0.595	0.307	0.449	0.560	0.331	0.446	0.553	0.863	–
λ	1.512	3.289	10.371	-0.121	1.992	3.904	0.180	1.918	4.416	0.835	0.273
$\lambda = 4$											
β_1	4.926	5.001	5.093	4.950	5.026	5.242	4.960	5.043	5.285	0.866	–
β_2	1.953	1.999	2.046	1.953	2.000	2.048	1.954	2.000	2.046	0.951	–
β_3	2.953	2.999	3.046	2.952	3.000	3.047	2.954	3.000	3.047	0.959	–
ω	0.407	0.495	0.585	0.305	0.460	0.553	0.335	0.456	0.555	0.883	–
λ	2.176	4.378	115.504	0.014	2.550	4.974	0.413	2.739	5.896	0.849	0.105
$\lambda = 5$											
β_1	4.933	5.000	5.069	4.955	5.019	5.118	4.963	5.029	5.212	0.918	–
β_2	1.957	1.998	2.042	1.957	1.998	2.043	1.958	1.998	2.042	0.964	–
β_3	2.958	2.998	3.041	2.957	2.998	3.044	2.957	2.999	3.041	0.971	–
ω	0.416	0.497	0.576	0.308	0.468	0.549	0.350	0.466	0.552	0.916	–
λ	2.882	5.844	135.799	0.119	3.197	6.105	0.906	3.579	7.721	0.895	0.027

Table 7S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-normal distribution, $n = 100$.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.567	4.976	5.436	4.613	5.000	5.367	4.774	4.996	5.237	0.994	–
β_2	1.945	2.002	2.052	1.945	2.001	2.053	1.946	2.001	2.052	0.954	–
β_3	2.945	3.001	3.054	2.942	3.000	3.054	2.945	3.000	3.054	0.941	–
ω	0.498	0.584	0.688	0.479	0.519	0.577	0.497	0.539	0.603	0.894	–
λ	-1.363	0.038	1.452	-0.912	-0.002	0.929	-0.609	0.001	0.612	0.993	1.166
$\lambda = 1$											
β_1	4.867	5.008	5.285	4.898	5.180	5.444	4.946	5.216	5.360	0.858	–
β_2	1.959	1.999	2.041	1.955	1.999	2.043	1.958	1.999	2.041	0.944	–
β_3	2.953	2.999	3.045	2.951	2.999	3.045	2.952	2.999	3.045	0.942	–
ω	0.396	0.493	0.589	0.394	0.430	0.554	0.410	0.450	0.540	0.968	–
λ	0.017	0.994	1.873	-0.281	0.108	1.556	-0.206	0.190	1.370	0.862	1.095
$\lambda = 2$											
β_1	4.931	5.004	5.107	4.941	5.021	5.313	4.949	5.040	5.300	0.857	–
β_2	1.964	2.001	2.036	1.961	2.001	2.037	1.963	2.001	2.034	0.951	–
β_3	2.965	3.001	3.034	2.963	2.999	3.035	2.965	3.000	3.035	0.962	–
ω	0.420	0.494	0.559	0.340	0.474	0.552	0.362	0.466	0.547	0.858	–
λ	1.145	2.000	3.017	-0.055	1.665	2.603	0.230	1.585	2.631	0.842	0.205
$\lambda = 3$											
β_1	4.951	5.003	5.063	4.958	5.011	5.081	4.962	5.016	5.098	0.932	–
β_2	1.970	2.000	2.031	1.969	2.000	2.032	1.970	2.000	2.031	0.964	–
β_3	2.968	3.000	3.030	2.966	3.000	3.031	2.968	3.000	3.031	0.956	–
ω	0.439	0.496	0.552	0.418	0.485	0.542	0.417	0.485	0.542	0.936	–
λ	2.080	3.067	4.780	1.633	2.583	3.845	1.600	2.685	4.076	0.917	5×10^{-9}
$\lambda = 4$											
β_1	4.957	4.999	5.047	4.962	5.007	5.058	4.965	5.009	5.059	0.953	–
β_2	1.974	2.000	2.027	1.973	2.000	2.028	1.974	2.000	2.027	0.968	–
β_3	2.972	3.000	3.025	2.971	3.000	3.025	2.972	3.000	3.026	0.974	–
ω	0.446	0.500	0.548	0.435	0.491	0.540	0.436	0.492	0.540	0.949	–
λ	2.861	4.222	6.612	2.396	3.487	5.141	2.485	3.648	5.404	0.946	8×10^{-13}
$\lambda = 5$											
β_1	4.966	5.001	5.044	4.972	5.007	5.053	4.974	5.009	5.056	0.945	–
β_2	1.973	2.000	2.025	1.973	2.000	2.025	1.974	2.000	2.025	0.959	–
β_3	2.974	2.999	3.024	2.974	3.000	3.025	2.974	3.000	3.024	0.961	–
ω	0.452	0.497	0.546	0.443	0.489	0.535	0.443	0.490	0.538	0.956	–
λ	3.481	5.202	8.767	2.888	4.263	6.408	3.048	4.488	6.859	0.937	7×10^{-13}

Table 8S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-normal distribution, $n = 250$.

Method	MLE			MAP			Median			Coverage	BF
	5%	50%	95%	5%	50%	95%	5%	50%	95%		
$\lambda = 0$											
β_1	4.617	4.994	5.379	4.642	5.004	5.354	4.776	4.997	5.228	0.994	–
β_2	1.962	2.000	2.039	1.960	1.999	2.040	1.961	2.000	2.040	0.939	–
β_3	2.963	3.001	3.037	2.960	3.001	3.041	2.962	3.001	3.037	0.948	–
ω	0.501	0.568	0.648	0.485	0.514	0.555	0.499	0.532	0.587	0.884	–
λ	-1.142	0.003	1.140	-0.952	-0.010	0.954	-0.590	0.005	0.602	0.994	1.276
$\lambda = 1$											
β_1	4.900	5.000	5.267	4.910	5.067	5.400	4.943	5.178	5.319	0.826	–
β_2	1.970	2.001	2.030	1.968	2.000	2.031	1.969	2.001	2.030	0.958	–
β_3	2.970	2.999	3.031	2.969	2.998	3.030	2.969	2.999	3.030	0.951	–
ω	0.405	0.499	0.562	0.402	0.429	0.551	0.414	0.448	0.535	0.927	–
λ	0.047	1.014	1.561	-0.307	0.360	1.408	-0.105	0.327	1.314	0.813	1.075
$\lambda = 2$											
β_1	4.947	5.003	5.065	4.952	5.010	5.081	4.956	5.016	5.114	0.916	–
β_2	1.976	2.001	2.025	1.976	2.001	2.026	1.976	2.001	2.025	0.964	–
β_3	2.975	3.000	3.024	2.974	3.000	3.026	2.975	3.000	3.024	0.955	–
ω	0.445	0.497	0.546	0.419	0.490	0.542	0.417	0.488	0.540	0.910	–
λ	1.436	1.983	2.682	1.227	1.826	2.488	1.070	1.837	2.530	0.913	1×10^{-17}
$\lambda = 3$											
β_1	4.962	5.000	5.042	4.963	5.004	5.049	4.967	5.006	5.051	0.945	–
β_2	1.977	1.999	2.020	1.977	2.000	2.020	1.978	1.999	2.020	0.957	–
β_3	2.979	3.000	3.021	2.978	3.000	3.022	2.979	3.000	3.021	0.962	–
ω	0.461	0.500	0.537	0.452	0.494	0.535	0.454	0.495	0.535	0.944	–
λ	2.325	3.050	4.081	2.113	2.810	3.691	2.162	2.876	3.791	0.938	5×10^{-53}
$\lambda = 4$											
β_1	4.968	5.001	5.033	4.970	5.004	5.038	4.972	5.005	5.039	0.945	–
β_2	1.982	2.000	2.020	1.982	2.001	2.020	1.982	2.000	2.020	0.972	–
β_3	2.979	3.000	3.019	2.979	3.000	3.020	2.980	3.000	3.019	0.949	–
ω	0.461	0.499	0.532	0.457	0.494	0.528	0.457	0.495	0.528	0.952	–
λ	3.064	4.049	5.470	2.826	3.700	4.981	2.879	3.796	5.074	0.934	1×10^{-52}
$\lambda = 5$											
β_1	4.976	5.000	5.028	4.978	5.003	5.033	4.979	5.004	5.033	0.956	–
β_2	1.981	2.000	2.019	1.981	2.000	2.019	1.981	2.000	2.018	0.951	–
β_3	2.982	3.000	3.017	2.981	3.000	3.018	2.982	3.000	3.017	0.959	–
ω	0.464	0.497	0.532	0.460	0.494	0.528	0.461	0.494	0.529	0.943	–
λ	3.877	5.086	7.157	3.514	4.626	6.353	3.601	4.729	6.475	0.947	4×10^{-48}

Table 9S. Point estimators, coverage proportions and Bayes factors: Linear regression model with residual errors simulated from a skew-normal distribution, $n = 500$.